PH458

Aggregating Judgment

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Introduction

Social choice theory

- This is the study of how to aggregate or combine individual opinions, preferences and decisions.

- Actually, it's not one theory but a general framework within which various theories and/or models are subsumed.

    Domains of application:
    * economics
    * institutional design
    * political theory
    * …

    NB: This topic relates nicely to the topic of testimony and in particular how to combine multiple testimonies.

What is an aggregation rule?

- An **aggregation rule** is a function that takes individual votes as inputs and gives a collective vote as output.

  Each individual $i \in N$ containing n of them votes v_i in relation to 2 options: $v_i = 1$ (opt 1); $v_i = -1$ (opt 2); $v_i = 0$ (abstention)

  For each combination of votes $<v_1, v_2, \ldots v_n>$:

Majority rule:

$$f_m = \begin{cases} 1 \text{ if } v_1 + v_2 + \ldots + v_n > 0 \\ 0 \text{ if } v_1 + v_2 + \ldots + v_n = 0 \\ -1 \text{ if } v_1 + v_2 + \ldots + v_n < 0 \end{cases}$$

Weighted majority rule:

$$f_{wm} = \begin{cases} 1 \text{ if } w_1 v_1 + \ldots + w_n v_n > 0 \\ 0 \text{ if } w_1 v_1 + \ldots + w_n v_n = 0 \\ -1 \text{ if } w_1 v_1 + \ldots + w_n v_n < 0 \end{cases}$$

Other aggregation rules

- Without articulating their functions, we can articulate the following (inexhaustive) list of rules:

  * unanimity

  * symmetrical supermajority

  * asymmetrical supermajority

  * sub-majority

  * dictatorship

Social choice theory and policy

- Social choice theory (SCT) can also be described as the study of how we convert individual inputs into collective outputs.

- Such conversions are important in policy design where many voices must be combined into one.

  Sample question:
  How do we arrive at rational collective judgments from the rational individual judgments of various experts?

- Special problems arise in trying to make such conversions. SCT uncovers such problems and offers ways to solve them.

Some Results

Condorcet

Nicolas de Condorcet

(1743-1794)

- One of the founding fathers of social choice theory.

- Liberal thinker who examined the consequences of voting systems.

- To be exact: the relation between probability and majority decisions.

- Proposed two antithetical results: (1) a jury theorem & (2) a paradox.

Condorcet jury theorem

- **The theorem**:
  "... if each member of a jury has an equal and independent chance better than random, but worse than perfect, of making a correct judgment on whether a defendant is guilty (or on some other factual proposition), the majority of jurors is more likely to be correct than each individual juror, and the probability of a correct majority judgment approaches 1 as the jury size increases" (List 2013, p. 2).

- In short, under certain conditions, majority voting is good at tracking the truth.

- More precisely, it is better at tracking the truth than a randomly chosen individual.

Condorcet's paradox

- **The paradox:**

  Aggregating rational individual preferences via a majority rule may result in irrational collective preferences.

- Why is this paradoxical?

- Because, pace CJT, majority rule seems plausible but, pace some assumptions, it's not only implausible but impossible!

Condorcet's paradox: Example

- Transitivity is a property of relations. It is possessed when a relation satisfies the following rule:

  Transitivity rule: For any *x*, *y* and *z*, if *Rxy* and *Ryz* then *Rxz*.

- Suppose 3 groups have the following preferences: *A* prefers φ to χ to ψ; *B* prefers χ to ψ to φ; *C* prefers ψ to φ to χ.

- 2/3 majorities emerge of (i) φ to χ, (ii) χ to ψ and (iii) ψ to φ.

- These violate transitivity for we have *P$\varphi\chi$* and *P$\chi\psi$* but instead of *P$\varphi\psi$* we have *P$\psi\varphi$*.

  where 'P' stands for 'prefers over'.

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Arrow's impossibility theorem

- Taught and influenced by Alfred Tarski, he won the 1972 Economics Nobel prize.

- He built on and generalised Condorcet's work proving an impossibility theorem.

- Such theorems show that otherwise plausible suppositions are inconsistent.

- He showed that a *class* of aggregation methods cannot satisfy plausible axioms.

Kenneth Arrow

(1921-2017)

Arrow's impossibility theorem (2)

- In a nutshell, the theorem states:

  When 2 or more agents rank 3 or more preferences, those cannot be converted into collective preferences via *M*.

  where *M* is a method that satisfies five, at first glance, plausible suppositions:

  * Universal domain
  * Non-dictatorship
  * Ordering
  * Weak pareto principle
  * Independence of irrelevant alternative

- The kinds of scenarios studied by social choice theory often involve such breakdowns.

- When do they occur?

  * When two (sometimes three) or more individuals vote
  * The vote is on a set of logically related propositions
  * Each individual's vote is rational [blue]
  * The tally exhibits split majorities [red]
  * The split majorities violate a logical rule

  Hence, the majority judgements are irrational because they do not satisfy the logical rules employed.

- "[S]uppose that a three-judge court... must judge the defendant liable *if and only if* it finds, first, that the defendant's negligence was causally responsible for the injury... and, second, that the defendant had a duty of care... Suppose that the three judges, A, B, and C, vote as shown below..." (Goldman 2006, p. 13).

- Aggregating via majority means the first 2 columns are 'Yes' but the 3rd comes out 'No'.

	Cause of harm?	Duty of care?	Liable?
A	Yes	No	No
B	No	Yes	No
C	Yes	Yes	Yes
Majority	Yes	Yes	No

- **Upshot**: This contradicts the above rule, namely that two 'Yesses' make a 'Yes'.

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Why care about such results?

- Because they demonstrate how two of our most cherished cultural developments come into conflict.

  Democracy seeks to establish legitimacy on the basis of some form of equal representation and hence anti-minority rule.

  Rationality seeks to establish coherent beliefs on the basis of reasoning norms.

- The two need not always conflict. In fact, some theorems emerge when they're in agreement, e.g. May (1952):

  An aggregation rule f satisfies anonymity, neutrality, universal domain and positive responsiveness IFF f is majority rule.

Side-stepping Impossibilities

To drop or relax?

- Theorems are theorems! There is no getting around them if you start from the *same* assumptions.

- But we can of course either drop those assumptions altogether or relax them.

- In what follows, we consider precisely such tactics.

Example: Universal domain

- **Universal Domain**: All logically possible combinations of choices are allowed.

  Intuitively: 'Freedom of thought!'

- We can relax this condition by restricting the admissible combinations of choices, e.g. by imposing a no-tie rule!

- There are obvious positive and negative side-effects.

  <u>positive</u>: in urgent action cases, decisions can be reached

  <u>negative</u>: decisions may be forced

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Example: Anonymity

- **Anonymity**: "The decision procedure is invariant under permutations of the individuals" (List 2006: 376).

  Intuitively: 'Everyone matters as much!'

- We can relax this condition by assigning different weights to different individuals, e.g. in accordance with expertise.

- There are obvious positive and negative side-effects.

  <u>positive</u>: more reliable individuals can have more of say

  <u>negative</u>: collective decision may be rigged for personal gain

Private and Public
Decision Procedures

- **Premise-driven**: The group's opinion is settled by aggregation on the premises, then we 'let logic decide' (Pettit 2003).

  Advantage: Allows us to cite reasons and relies on the safety of logic to reach conclusions. Cf. List (2006).

- **Conclusion-driven**: The group's opinion is settled by aggregation on the conclusion alone.

  Advantage: Group opinion on C is decided by simply consulting each individual members' opinion on C.

The discursive dilemma

- "The premise- and conclusion-based procedures clearly differ in whether or not any collective judgments are made on the premises, but they may also differ in the collective judgment they generate on the conclusion, as shown next. This problem has become known as the "doctrinal paradox" (Kornhauser and Sager 1986, 1993) or "discursive dilemma" (Pettit 2001a) and has sparked a technical literature on judgment aggregation" (List 2006: 369).

Two approaches in conflict – a conjunctive case

- The conclusion-driven approach:

	P_1	P_2	$(P_1 \& P_2) \leftrightarrow C$	C
Person 1	True	True	True	True
Person 2	True	False	True	False
Person 3	False	True	True	False
Majority				**False**

- The premised-driven approach:

	P_1	P_2	$(P_1 \& P_2) \leftrightarrow C$	C
Person 1	True	True	True	
Person 2	True	False	True	
Person 3	False	True	True	
Majority	True	True	True	**True**

Example: Tenure

- The conclusion-driven approach:

	Exc Teaching	Exc Research	$(P_1 \ \& \ P_2) \leftrightarrow C$	Tenure
Person 1	True	True	True	True
Person 2	True	False	True	False
Person 3	False	True	True	False
Majority				**False**

- The premised-driven approach:

	Exc Teaching	Exc Research	$(P_1 \ \& \ P_2) \leftrightarrow C$	Tenure
Person 1	True	True	True	
Person 2	True	False	True	
Person 3	False	True	True	
Majority	True	True	True	**True**

Two accounts of collective decision

- **Minimal Liberal (ML)**: Collective decisions should be made only on outcomes; reasons should be kept private.

- **Comprehensive Deliberative (CD)**: Collective decisions should also include reasons (and even logical relations).

- There are obvious similarities between ML and conclusion-based as well as CD and premise-based accounts.

  * ML: reasons should be kept private
  * C-B: individual's conclusion derived from their premises

  * CD: reasons should be publicly deliberated
  * P-B: aggregation on the premises

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Simple and nonsimple public agendas

- Public agendas: sets of propositions for collective decision.

- List (2006) identifies two kinds:

  Simple: "if it includes only a single 'atomic' proposition such as C (or multiple unconnected ones)" (p. 373)
  Nonsimple: "if it includes two or more 'atomic' propositions ... and at least one 'compound' proposition... for representing logical relations" (p. 373).

- He then argues that impossibility results (including two of his own) affect nonsimple public agendas.

- Since CD *generally* underwrites nonsimple agendas but ML doesn't, the former fares worse than the latter.

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The End